

# Scalar Form Factors and Nuclear Interactions

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**Abstract.** The scalar-isoscalar term in the two-pion exchange  $NN$  potential is abnormally large and does not respect the hierarchy of effects predicted by chiral perturbation theory. We argue that this anomaly is associated with non-perturbative effects, which are also present in the  $\pi N$  scalar form factor.

## 1 Nuclear Potentials

In the last fifteen years, chiral perturbation theory (ChPT) has been systematically employed in the study of nuclear interactions. In ChPT, amplitudes are expanded in power series of a typical variable  $q$ , representing either pion four-momenta or nucleon three-momenta, such that  $q \ll 1$  GeV. In the case of nuclear interactions, ChPT predicts a hierarchy in which potentials have structures with different orders of magnitude.

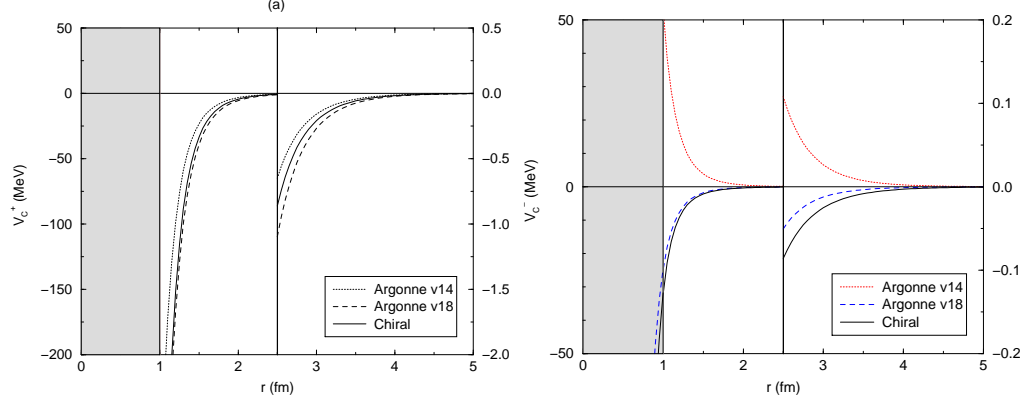
The leading contribution to the  $NN$  potential begins at  $\mathcal{O}(q^0)$  and is due to the *OPEP*[1]. The two-pion exchange contribution (*TPEP*) begins at  $\mathcal{O}(q^2)$  and has already been expanded up to  $\mathcal{O}(q^4)$ , by means of both heavy baryon[2] and covariant[3] ChPT. The leading contribution to the three-nucleon force, associated with two-pion exchange, begins at  $\mathcal{O}(q^3)$  and  $\mathcal{O}(q^4)$  corrections are presently being evaluated[4]. Nowadays, about 20 nuclear-force components are known and the overall picture can be assessed.

An outstanding problem in the hierarchy predicted by ChPT concerns the relative sizes of the isospin independent ( $V_C^+$ ) and dependent ( $V_C^-$ ) central components of the *TPEP*[5], displayed in Fig.1. According to ChPT, the former begins at  $\mathcal{O}(q^3)$  and the latter at  $\mathcal{O}(q^2)$ . On the other hand, the figure shows that the chiral hierarchy is defied, since  $V_C^+ \sim 10 |V_C^-|$ . These profile functions were scrutinized in Ref.[5] and the function  $V_C^+$  was found to be heavily dominated by a term of the form

$$V_C^+ \sim - (4/f_\pi^2) \left[ (c_3 - 2c_1) - c_3 \nabla^2/2 \right] \tilde{\sigma}_{NN} , \quad (1)$$

where the  $c_i$  are LECs and  $\tilde{\sigma}_{NN}$  is the leading contribution of the pion cloud to the nucleon scalar form factor. This close relationship between  $\tilde{\sigma}_{NN}$  and  $V_C^+$

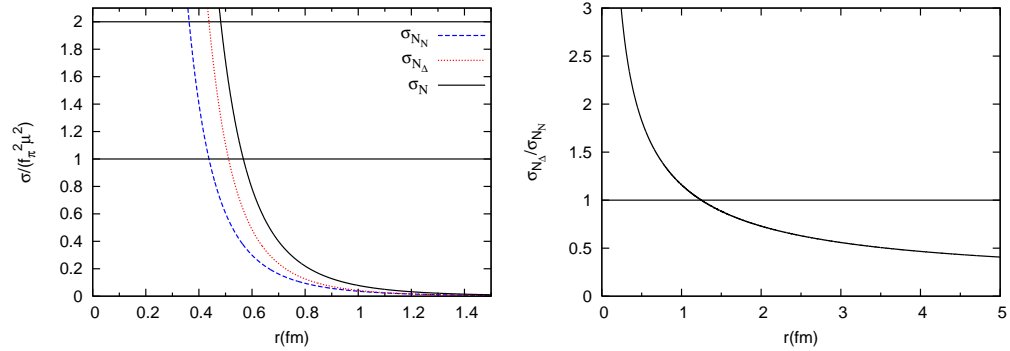
indicates that the study of the former can shed light into the properties of the latter.



**Figure 1.** Isospin independent ( $V_c^+$ ) and dependent ( $V_c^-$ ) central components of the *TPEP*[5].

## 2 Scalar Form Factor

The discussion in this section is based on results derived in Ref.[6]. The nucleon scalar form factor is defined as  $\langle N(p') | -\mathcal{L}_{sb} | N(p) \rangle = \sigma_N(t) \bar{u}(p') u(p)$ , where  $\mathcal{L}_{sb} = -\hat{m}(\bar{u}u + \bar{d}d)$  is the symmetry breaking lagrangian. The function  $\sigma_N(t)$  can be well represented by a leading  $\mathcal{O}(q^2)$  tree contribution associated with the LEC  $c_1$ , supplemented by two triangle diagrams, involving nucleon and delta intermediate states, which give rise respectively to  $\mathcal{O}(q^3)$  and  $\mathcal{O}(q^4)$  corrections. In configuration space contact and loop contributions split apart and one has  $\tilde{\sigma}_N(\mathbf{r}) = -4c_1\mu^2\delta^3(\mathbf{r}) + \tilde{\sigma}_{N_N}(r) + \tilde{\sigma}_{N_\Delta}(r)$ .



**Figure 2.** Ratios  $\tilde{\sigma}_N(r)/(\mu^2 f_\pi^2) = (1 - \cos \theta)$  (left) and  $\tilde{\sigma}_{N_\Delta}(r)/\tilde{\sigma}_{N_N}(r)$  (right).

The symmetry breaking lagrangian is written in terms of the chiral angle  $\theta$  as  $\mathcal{L}_{sb} = f_\pi^2 \mu^2 (\cos \theta - 1)$ , in a convention in which the energy density vanishes when  $r \rightarrow \infty$  ( $\mu$  is the pion mass). The ratio  $\tilde{\sigma}_N(\mathbf{r})/(\mu^2 f_\pi^2) = (1 - \cos \theta)$  is displayed in Fig.2(left) and increases monotonically as one approaches the center of the nucleon. On the other hand, the physical interpretation of the quark

condensate corresponds to the condition  $\langle 0|q\bar{q}|0\rangle > 0$  and the function  $\tilde{\sigma}_N(\mathbf{r})$  becomes meaningless beyond a critical radius  $R$ , corresponding to  $\theta = \pi/2$ . In Ref.[6] we assumed that the condensate no longer exists in the region  $r < R$  and evaluated the  $\pi N$  sigma-term using the expression

$$\sigma_N = \frac{4}{3}\pi R^3 f_\pi^2 \mu^2 + 4\pi \int_R^\infty dr r^2 \tilde{\sigma}_N(\mathbf{r}). \quad (2)$$

Depending on the value adopted for the  $\pi N \Delta$  coupling constant, this procedure yielded the range[6]  $43 \text{ MeV} < \sigma_N < 49 \text{ MeV}$ , which is fully compatible with the empirical value  $45 \pm 8 \text{ MeV}$ . This indicates that our picture of the nucleon scalar form factor is sound and can be used to gain insight about  $V_C^+$ .

### 3 Conclusions

In the case of the scalar form factor, contributions from  $N$  and  $\Delta$  intermediate states are respectively  $\mathcal{O}(q^3)$  and  $\mathcal{O}(q^4)$ . Inspecting Fig.2, one learns the hierarchy predicted by ChPT is subverted for distances smaller than 1.5 fm, where the delta becomes more important than the nucleon. On the other hand, the good results obtained from eq.(2) for the nucleon sigma-term (and also for  $\sigma_\Delta$ , [6]) indicate that the functions  $\tilde{\sigma}_{N_N}(r)$  and  $\tilde{\sigma}_{N_\Delta}(r)$  convey the correct physics at least up to the critical radius  $R \sim 0.6 \text{ fm}$ , which lies well beyond the domain of the validity of ChPT. Just outside this radius the chiral angle is of the order of  $\pi/2$ , a clear indication of the non-perturbative character of the problem.

This finding is supported by the results from the study of the  $TPEP$  performed in Ref.[5]. In that work the predictions for  $V_C^+$ , reproduced in our Fig.1(left), were in rather close agreement with the Argonne phenomenological potentials[7] up to 1 fm, in spite of the fact that power counting was ineffective already at 2 fm, as shown in their Fig.6.

Our main conclusion, which needs to be further tested, is that the range of validity of calculations based on nucleon and delta intermediate states is much wider than that predicted by ChPT.

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